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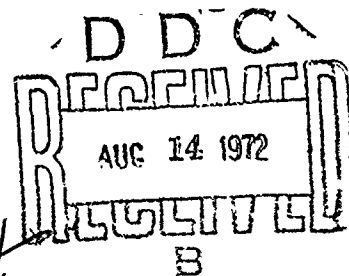
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13. ABSTRACT

Often in process simulation studies and process control it is desirable to have an approximate transfer function that could be used for dynamic simulation via analog computer or for feedforward control algorithms.

The scope of this work is concerned with developing a general analytical technique that can be easily implemented in obtaining approximate transfer functions for distributive parameter systems of the convective type, where the spacial dependence of the dependent variable is of little interest from a control viewpoint.

This technique relies on a distributed parameter model that is transformed into ordinary differential equations in the frequency domain. These equations are then analyzed in the frequency domain using well known classical techniques. The frequency solution of the ordinary differential equation provides the necessary data to carry out regressions on the parameters of the postulated transfer function. After an acceptable fit is obtained in one or more of the transfer functions, they are inverted into the time domain and compared against the numerical solution of the model equation.
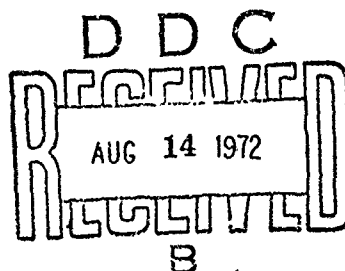
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Investigation of a
Frequency Domain Identification Technique
for Distributed Parameter System

Technical Report No. 47
LSU-T-TR-47

by

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Summary

The frequency response of a shell and tube heat exchanger was obtained by the solution of an ordinary differential equation in the frequency domain. This equation was obtained by a Fourier transformation of the set of partial differential equations representing the dynamic time response of the system.

Three simple transfer functions were evaluated for their ability to approximate the dynamics of the exchanger. The transfer function

$$\frac{(1 - Ke^{-tos})}{(\tau_1 s + 1)(\tau_2 s + 1)}$$

was found to approximate the system dynamics exceptionally well.

A method of evaluating the parameters of the approximate transfer function by relating the frequency response to the time response via Parraval's theorem was also evaluated. Little advantage was found in this method.

INTRODUCTION

Often in process simulation studies and process control it is desirable to have an approximate transfer function that could be used for dynamic simulation via analog computer or for feedforward control algorithms.

In many cases it is only desired to know the system response to some disturbance at a fixed space corrdinate, even though the system is space dependent. For example, if the system under consideration is a plug flow reactor, the reaction occurs continuously along the length of the reactor causing reactant and product concentrations to be space dependent. Perhaps it is necessary to know how changes in the feed composition affect product distribution. This being the case an approximate transfer function relating product distribution dynamics to changes in feed composition could be employed in a feedforward control strategy.

Systems whose dynamic response are space dependent are frequently referred to as distributive parameter systems. The equations describing their dynamic behavior are partial differential equations in time and one or more space coordinates. Classical time domain solutions for these equations are very hard to come by, and often they are considered a hopeless undertaking.

The scope of this work is concerned with developing a general analytical technique that can be easily implemented in obtaining approximate transfer functions for distributive parameter systems of

the convective type, where the spacial dependence of the dependent variable is of little interest from a control viewpoint.

This technique relies on a distributed parameter model that is transformed into ordinary differential equations in the frequency domain. These equations are then analyzed in the frequency domain using well known classical techniques. The frequency solution of the ordinary differential equation provides the necessary data to carry out regressions on the parameters of the postulated transfer function. After an acceptable fit is obtained in one or more of the transfer functions, they are inverted into the time domain and compared against the numerical solution of the model equation.

Basic Theory

The equations describing the dynamics of distributive parameter systems are partial differential equations in time and one or more space coordinates. Classical time domain solutions of these equations have been found only for a few cases of practical interest.

It has been known for many years that linear partial differential equations can be analyzed in the frequency domain provided the system dynamics are described at a fixed position of the spacial coordinates. Although restricting the analysis to a fix point in the spacial coordinates presents no serious drawbacks, the complex frequency expressions that result in most applications are not suitable for time domain inversion with classical methods. The ever increasing availability of digital computers coupled to the development of sophisticated non-linear regressing techniques have provided the necessary tools to make the time domain inversion possible.

In this section the theoretical aspects involved in the frequency analysis of the partial differential equation and the time domain inversion of the resulting frequency expression will be discussed.

To translate the partial differential equation to the frequency domain, it is necessary to obtain the Laplace transform of the individual terms in the equation.

The Laplace transform of a continuous and differentiable space dependent variable is defined as:

$$F(x,s) = \int_0^\infty f(x,t) \exp(-ts) \, dt \tag{1}$$

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By using equation (1) to obtain the Laplace transform of $\frac{\partial f(x,t)}{\partial t}$ one obtains:

$$\left\{\frac{\partial f(x,t)}{\partial t}\right\} = \int_0^\infty \frac{\partial f(x,t)}{\partial t} \exp(-ts)dt \tag{2}$$

Integrating equation (2) by parts gives:

$$\int_0^\infty \frac{\partial f(x,t)}{\partial t} \exp(-ts)dt = f(x,t)\exp(-ts) \Big|_0^\infty + s \int_0^\infty f(x,t)\exp(-st)dt \tag{3}$$

Evaluating limits and using equation (1) to obtain:

$$\left\{\frac{\partial f(x,t)}{\partial t}\right\} = sF(x,s)-f(x,o) \tag{4}$$

The Laplace transform of higher order partial derivatives of $f(x,t)$ with t are evaluated in the same manne   The nth order partial derivative is given by equation (5):

$$\left\{\frac{\partial f(x,t)}{\partial t^n}\right\} = s^n F(x,s)-s^{n-1}f(x,o)-s^{n-2}\frac{\partial f(x,o)}{\partial t} \tag{5}$$

$$-\ldots- \frac{\partial^{n-1}}{\partial t^{n-1}} f(x,o)$$

The Laplace transform of $\frac{\partial f(x,t)}{\partial x}$ is evaluated as follows:

$$F(x,s) = \int_0^\infty f(x,t)\exp(-ts)dt$$

$$\frac{\partial F(x,s)}{\partial x} = \frac{\partial}{\partial x} \int_0^\infty f(x,t)\exp(-ts)dt = \int_0^\infty \frac{\partial}{\partial x} [f(x,t)\exp(-ts)]dt$$

or

$$\frac{\partial F(x,s)}{\partial x} = \int_0^\infty \frac{\partial f(x,t)}{\partial x} \exp(-ts)dt \tag{6}$$

A comparison between equations (1) and (6) will show that

$$\left\{\frac{\partial f(x,t)}{\partial x}\right\} = \frac{\partial F(x,s)}{\partial x} \tag{7}$$

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Higher order derivatives are handled in an analogous manner.

The Laplace transform of the nth order partial of $f(x,t)$ with x is:

$$\left\{\frac{\partial^n f(x,t)}{\partial t^n}\right\} = \frac{\partial^n F(x,s)}{\partial x^n} \tag{8}$$

Equations (5) and (8) can be used to obtain the Laplace transform of mixed and higher order partial derivatives as shown by Haberman (8).

Having completed the frequency analysis of the partial differential equation, the lumped parameter transfer function must then be inverted into the time domain so as to obtain the desired time domain representation. It was mentioned earlier that in most cases the lumped parameter transfer function cannot be readily inverted by classical methods. Thus the problem is one of selecting a suitable functional form for the transfer function and secondly to "fit" the transfer function to the frequency response data such that there is good agreement in the time domain (2).

Little can be said about selecting a suitable transfer function, other than it must be readily inverted into the time domain and should include terms characteristic of the response of the particular system.

To obtain the best statistical agreement in the time domain, the following criteria function was chosen for minimizations:

$$SSDRT = \int_o^{t_{st}} \left\{g(x,t) - h(x,t)\right\}^2 dt \tag{9}$$

where

$g(x,t)$ = actual time function of the system

$h(x,t)$ = approximate time function of the system

t_{st} = settling time

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Through an application of Parseval's Theorem, Schnelle (3) has shown that the frequency domain equivalent of Equation (9) is:

$$\text{SSDRF} = \int_0^\infty [G^2 - 2GH\cos(\alpha - \phi) + H^2]d\omega \tag{10}$$

where

$G = |f(x,\omega)|$ = actual magnitude of system frequency response

$H = |g(x,\omega)|$ = approximate magnitude of system frequency response

α = actual phase angle of system frequency response

ϕ = approximate phase angle of system frequency response

Thus to obtain the best time domain representation by the minimizations of Equation (9), the integral in Equation (10) should be minimized when working with data in the frequency domain.

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Development of Model Equation

A relatively simple system that could be used to demonstrate the application of this method to distributive parameter systems of the convective type is a heat exchanger in which the temperature on one side of the tube wall is constant.

The particular system chosen consists of a one tube-pass, one shell-pass steam-hydrocarbon heat exchanger with the condensing steam on the shell side. (5)

The heat duty on the exchanger will be sensitive to changes in the condensing pressure of the steam. hydrocarbon flow rate and hydrocarbon inlet temperature.

A transfer function relating the liquid hydrocarbon outlet temperature to condensing temperature of the steam will be developed for the particular system just described and pictured below.
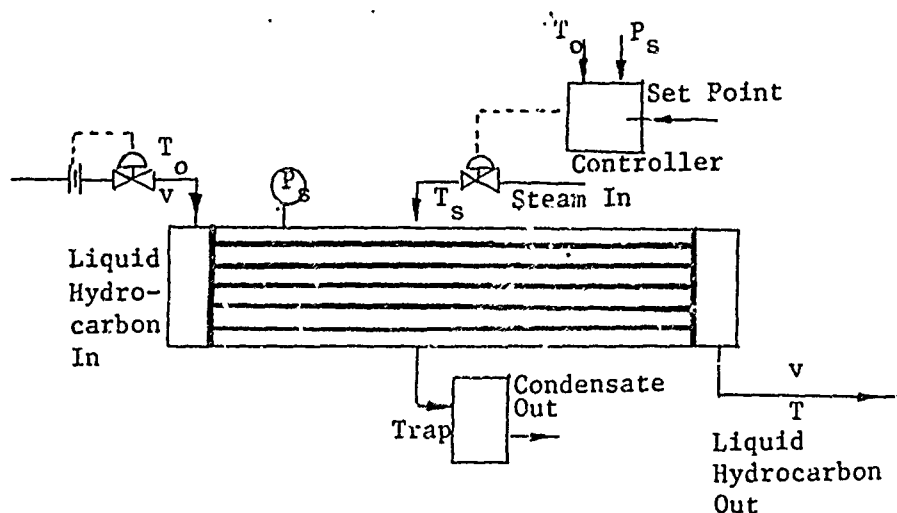

Figure 1. Pictorial representation of convective system under study.

The interactions between the shell wall capacity and tube wall
capacity are neglected. Also the lag in the steam pressure to the
valve opening due to the capacitance of the shell wall and vapors
within the shell is not considered. In order to maintain a constant
tube wall temperature throughout the length of the exchanger, it is
necessary to assume that the tube wall has no resistance and negli-
gible capacitance in the condensate film when compared with the
capacity of the liquid hydrocarbon film. The more common assumptions
of constant physical properties, no backmixing and constant liquid
hydrocarbon velocity are also employed.

By making use of these assumptions, the resulting convective
equation is:

$$T1 \frac{\partial T(x,t)}{\partial t} + T1v \frac{\partial T(x,t)}{\partial x} = T_w(t) - T(x,t) \tag{11}$$

where

$$T1 = \frac{A_{ic}\rho C_f}{h_i A_{is}} \quad (1/sec) \tag{12}$$

and the wall equation:

$$T2 \frac{dT_w(t)}{dt} + (T3+1)T_w(t) = T_s + T3T(x,t) \tag{13}$$

where

$$T2 = \frac{M_w C_w}{h_o A_{so}} \quad (1/sec) \tag{14}$$

$$T3 = \frac{h_i A_{si}}{h_o A_{so}} \quad (dimensionless) \tag{15}$$

Equations (11) and (13) complete the mathematical description of the system under study. These equations are coupled through the wall temperature indicating that a simultaneous solution is necessary if the system is to be solved.

Frequency Analysis

It is apparent that the simultaneous solution of Equations (11) and (13) via classical techniques would be very difficult to implement.

An alternative method for handling these simultaneous equations is by transformation into the frequency domain. The transformation from the time domain into the frequency domain reduces the wall equation to an algebraic equation and the convective equation to an ordinary differential equation.

It is convenient to define new variables representing deviations from the steady state value along any point within the exchanger before transformation of the equations.

$$\theta(x,t) = T(x,t) - T_{ss}(x) \tag{16}$$

$$\theta_w(t) = T_w - (T_w)ss \tag{17}$$

$$\theta_s(t) = T_s - (T_s)ss \tag{18}$$

After substituting above variables in Equations (11) and (13) and cancelling the steady state terms, the following equations result:

$$vT1 \frac{\partial\theta}{\partial x} + \frac{\partial\theta}{\partial t} = \theta_w - \theta \tag{19}$$

$$T2 \frac{d\theta_w}{dt} + (T3+1)\theta_w(s) + T3\theta(x,s) \tag{20}$$

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Transforming the above equations into the frequency domain one obtains:

$$vT1 \frac{d\theta(x,s)}{dx} + T1s\theta(x,s) = \theta_w(s) - \theta(x,s) \tag{21}$$

$$T2 \, s\theta_w(s) + (T3+1)\theta_w(s) + T3\theta(x,s) \tag{22}$$

Solving for $\theta_w(s)$ in equation (22):

$$\theta_w(s) = \frac{\theta_s(s) + T3\theta(x,s)}{T2s + (T3+1)}$$

Substituting in Equation (21) the expression just obtained for $\theta_w(s)$ results in

$$vT1 \frac{d\theta(x,s)}{dx} + \left(T1s - \frac{T3}{T2s+(T3+1)} + 1\right) \theta(x,s) = \frac{\theta_s(s)}{T2s+(T3+1)} \tag{23}$$

Multiplying equation (23) by $\dfrac{T2s+(T3+1)}{T1T2s^2+(T1+T2+T1T3)s+1}$ one obtains:

$$\frac{vT1(T2s+(T3+1)}{T1T2s^2+(T1+T2+T1T3)s+1} \frac{d\theta(x,s)}{dx} + \theta(x,s) = \frac{(T2s+(T3+1)) \, \theta_s(s)}{T1T2s^2+(T1+T2+T1T2)s+1} \tag{24}$$

Defining:

$$K1 = T1(T2+(T3+1)) \tag{25}$$

$$K2 = \frac{1}{T1T2s^2+(T1+T2+T1T3)s+1} \tag{26}$$

Equation (24) can be written in terms of K1 and K2 as:

$$K1K2v \frac{d\theta(x,s)}{dx} + \theta(x,s) = K2\theta_s(s) \tag{27}$$

The solution is easily obtained as:

$$\frac{\theta(x,s)}{\theta_s(s)} = K2\left(1-\exp\left(\frac{-x}{K1K2v}\right)\right) \tag{28}$$

Equation (28) is a transfer function relating liquid hydrocarbon temperature at any point within the exchanger to the temperature of the condensing steam. To obtain the desired transfer function, it is only necessary to lump the distributive liquid hydrocarbon temperature at the outlet of the exchanger. The desired transfer function relating the lumped liquid hydrocarbon temperature to changes in steam temperature is:

$$\frac{\theta(L,s)}{\theta_s(s)} = K2(1-\exp(\frac{-L}{K1K2v}))$$ (29)

The term $\frac{L}{K1K2v}$ contains an s^2 in the numerator and an s in the denominator, thus $\exp(\frac{-L}{K1K2v})$ is a vector with magnitude less than one and an ever-increasing phase lag. Consequently, the $(1-\exp(\frac{-L}{K1K2v}))$ will show regular fluctuations in magnitude and phase lag with frequency.

Even though it is very difficult to obtain an analytical expression in the time domain for Equation (29), an accurate representation of the system can still be obtained by approximating Equation (29) with other transfer functions that are readily inverted into the time domain.

In this work three different transfer functions were employed for this purpose. Non-linear regressions based on the frequency data generated by Equation (29) were used as a means to determine the respective parameters in the postulated transfer functions.

Computational Procedure

Upon completing the development of the model equation and its corresponding frequency solution, consideration must next be given to the computational techniques that will enable their numerical solution. The computational effort was twofold; one aimed at the solution of the model equation and the other aimed at determining an acceptable transfer function to approximate the frequency solution of the model equation.

Comparison of the time domain response of the approximate transfer functions to the numerical solution of the partial differential equations then provided a means of evaluating the postulated transfer function.

By a linear transformation of the existing variables and coordinates, equations (11) and (13) can be expressed in a more general form that at the same time is conducive to numerical calculations.

Defining a new set of temperatures of magnitude dictated by the arbitrary parameter T_a.

$$U(y, \Psi) = \frac{T(x,t) - T(o,t)}{T_a} \tag{30}$$

$$V(y, \Psi) = \frac{T_w(t) - T(o,t)}{T_a} \tag{31}$$

$$W(y, \Psi) = \frac{T_s(t) - T(o,t)}{T_a} \tag{32}$$

Defining now the new spacial and time coordinates through the usage of arbitrary parameters x_a and t_a.

$$y = \frac{x}{x_a} \tag{33}$$

$$\Psi = \frac{t}{t_a} \tag{34}$$

With the new set of temperatures a: l coordinates, the convective
and wall equations become:

$$\frac{T1}{T_a} \frac{\partial U(y,\Psi)}{\partial \Psi} + \frac{T1v}{x_a} \frac{\partial U(y,\Psi)}{\partial y} = V(\Psi) - U(y,\Psi) \tag{35}$$

$$\frac{T2}{T_a} \frac{dV(\Psi)}{d\Psi} + (T3+1)V(\Psi) - T3U(y,\Psi) = W(\Psi)$$

By choosing T_a to represent the algebraic difference between the
initial steam temperature and the inlet liquid hydrocarbon temperature,
the temperatures under consideration can be made dimensionless. To
normalize the spacial coordinates, x_a is chosen to be the length of the
exchanger. By assigning to t_a the numerical value of the time dependent
T2, the time coordinate is made dimensionless and at the same time a
slight simplification of the wall equation is accomplished. Equations
(36), (37) and (38) specify the value of the parameters just discussed.

$$T_a = T_s(o) - T(o,t) \tag{36}$$

$$x_a = L \tag{37}$$

$$t_a = T2 \tag{38}$$

By using the above values assigned to T_a, x_a and t_a, the con-
vective and wall equations become:

$$\frac{T1}{T2} \frac{\partial U(y,\Psi)}{\partial \Psi} + \frac{T1}{L} \frac{\partial U(y,\Psi)}{\partial y} = V(\Psi) - U(y,\Psi)$$

$$\frac{dV(\Psi)}{d\Psi} + (T4+1)V(\Psi) - T3U(y,\Psi) = W(\Psi)$$

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An abbreviated from of the above equation results by defining,

$$C_2 = T3+1 \tag{39}$$

$$C_3 = -T3 \tag{40}$$

$$C_4 = \frac{T1}{T2} \tag{41}$$

$$B = \frac{T1v}{L} \tag{42}$$

This form being

$$C_4 \frac{\partial U(y,\Psi)}{\partial \Psi} + B \frac{U(y,\Psi)}{\partial y} = V(\Psi)-U(y,\Psi) \tag{43}$$

$$\frac{dV(\Psi)}{d\Psi} + C_2 V(\Psi) + C_3 U(y,\Psi) = W(\Psi) \tag{44}$$

A numerical solution of Equations (43) and (44) was obtained using central difference analogs for the partial derivatives.

The steady state solution of the model equation was also obtained directly from equations (43) and (44) by eliminating the terms containing the derivative of the dimensionless time coordinate. The resulting equations possess an analytical solution given below:
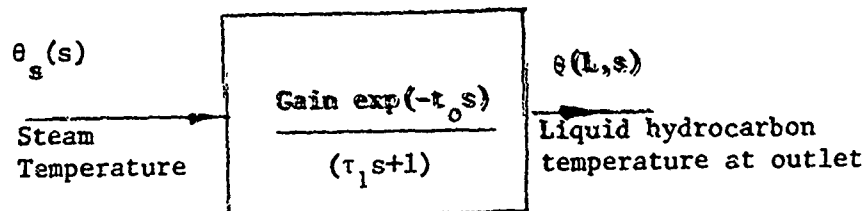
$$U(y,o) = W(o)(1-\exp(-C_1 y)) \tag{45}$$

where

$$C_1 = \frac{1}{BC_2}$$

$$W(o) = C_2 V(x) + C_3 U(y,o) \tag{46}$$
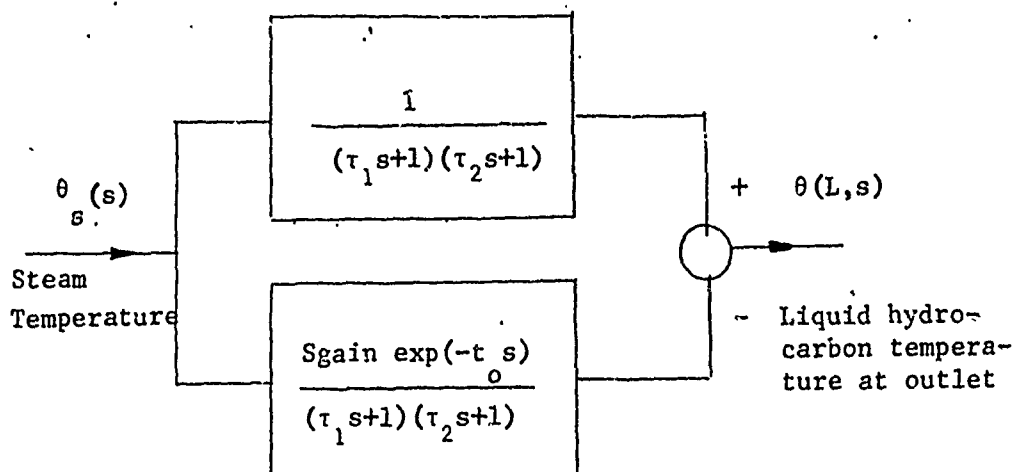
This initial distribution is required for the solution of the partial differential equations.

a) First order lag with dead time...$G_1(s)$
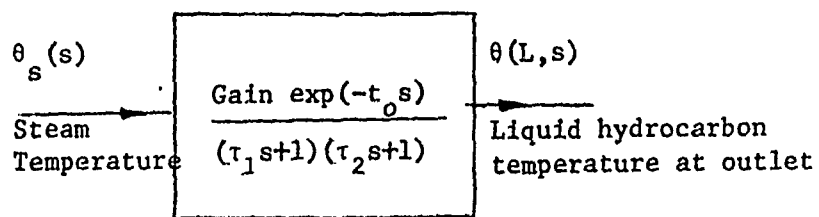
b) Second order lag with dead time...$G_2(s)$

c) Combination of a second order lag and a second order lag with dead time...$G_3(s)$

Figure 2. Postulated transfer functions used in analytical lumped parameter approximation technique

As mentioned earlier in this section, the other computational effort was aimed at obtaining an acceptable transfer function that approximates the frequency solution of the model equation (29). By using equation (29) to generate frequency data, non-linear regression analysis was carried out on the postulated transfer functions and the optimum parameters were thus obtained.

A program based on equation (29) was written to obtain the necessary frequency data for the regression analysis. A block diagram representation of each of the postulated transfer functions is shown in Figure 2.

For comparative purposes, the regression analysis was carried out on two objective functions. The first utilized the standard practice of obtaining the best parameter fit in the frequency domain by minimizing

$$PI = \int_0^\infty \{(G - H)^2 + (\theta - \phi)^2\}d\omega \qquad (47)$$

This will be referred to as frequency domain minimization.

The second objective function involved minimizing

$$\int_0^\infty \{g(x,t) - h(x,t)\}^2 dt$$

using frequency domain data by minimizing

$$PI = \int_0^\infty \{G^2 - 2GH \; Cos \; (\theta - \phi) + H^2\} \; d\omega \qquad (48)$$

as previously discussed. This second PI will be referred to as time domain minimization.

The rotational discrimination program developed by Farise and Law (4) was used to carry out the non-linear regressions. The program is based in a modified accelerated gradient technique, and is written in a self-calling subroutine structure.

For each transfer function, several regressions were carried out in both the frequency and time domain. Each regression was started at a different initial point with the hope of obtaining the best set of parameters.

After obtaining the best possible set of parameters for each of the postulated transfer functions, their frequency response was compared with that of equation (29). This comparison allowed the determination of the transfer function that best represented the frequency solution of the model equation. Upon selecting the postulated transfer function that best approximated equation (29), the final check to the analytical lumped parameter approximation technique was accomplished by inverting this transfer function into the time domain and comparing it with the numerical solution of the model equation.

To compare time-domain regressions against frequency regressions, the postulated transfer function was inverted into the time domain using the optimum set of parameters from both the frequency and time-domain regressions. For each transfer function its time domain representation is compared against the numerical solution of the model equation by first using the frequency regressed parameters and then the time-domain regressed parameters.

Results

Numerical Solution of Model Equation: The system was excited
with a 10°F step change in the steam temperature. The numerical
integration was performed with several $\Delta Y/\Delta T$ ratios, with a ratio
of 0.05 (0.005/0.10) giving the best results.

Although no stability problems were encountered, a slight dis-
crepancy (in the fifth significant figure) was observed between the
steady state values of the liquid hydrocarbon temperature calculated
from the numerical solution and those calculated analytically from
the steady state equations. This discrepancy was due to the capaci-
tance of the hydrocarbon film causing the tube wall temperature to
show slight distributive tendencies. Table I shows the steady states
calculated from the numerical solution and those calculated from the
steady state equations.

The time domain response of the liquid hydrocarbon temperature
at the outlet of the exchanger is shown in Figure 3.

Frequency Analysis of Model Equation: The frequency response of
the model equation is shown in Figure 4. Note the resonance peaks
characteristic of this system occurred in the medium and high
frequencies ($\omega \geq 0.55$ rads/sec). From this data, thirty-five points
were selected for the non-linear regressions.

Based on the data selected from the frequency response of the
model equation, several starting points were used for each of the
postulated transfer functions with the hope of obtaining a good estimate
of the global optimum. Tables II and III show the unimodel character-
istics of the response surface resulting from the frequency-domain
regressions of $G_1(s)$ and $G_2(s)$. These two tables show the local

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TABLE I

STEADY STATES OF THE DISTRIBUTED

LIQUID HYDROCARBON TEMPERATURE

| Actual Position (Ft)	Initial Steady State		Final Steady State	
	Numerical Solution (°F)	Steady State Equation (°F)	Numerical Solution (°F)	Steady State Equation (°F)
0.0	70.000	70.000	70.000	70.000
0.4	74.444	74.434	74.596	74.589
0.8	78.818	78.800	79.119	79.107
1.2	83.123	83.098	83.573	83.556
1.6	87.361	87.331	87.956	87.934
2.0	91.532	91.497	92.271	92.246
2.4	95.638	95.599	96.519	96.491
2.8	99.679	99.638	100.700	100.671
3.2	103.656	103.615	104.815	104.786
3.6	107.571	107.530	108.867	108.837
4.0	111.425	111.384	112.855	112.826
4.4	115.218	115.179	116.781	116.753
4.8	118.951	118.915	120.645	120.620
5.2	122.626	122.594	124.449	124.427
5.6	126.243	126.216	128.193	128.175
6.0	129.803	129.782	131.879	131.865
6.4	133.307	133.292	135.508	135.498
6.8	136.757	136.749	139.079	139.074
7.2	140.152	140.152	142.595	142.596
7.6	143.493	143.502	146.056	146.063
8.0	146.782	146.801	149.463	149.477
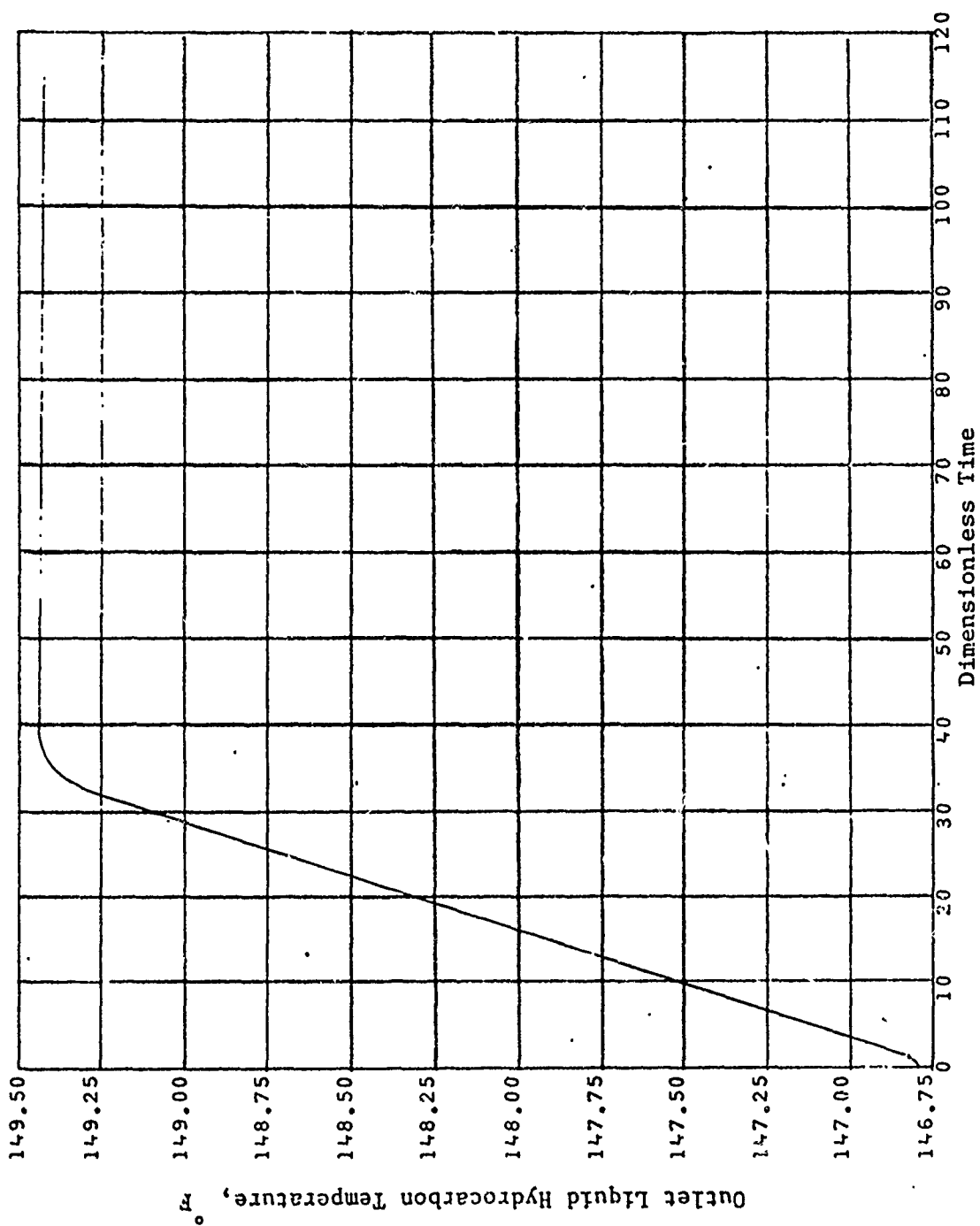
Figure 3. Actual time response of the liquid hydrocarbon temperature at outlet of exchanger.
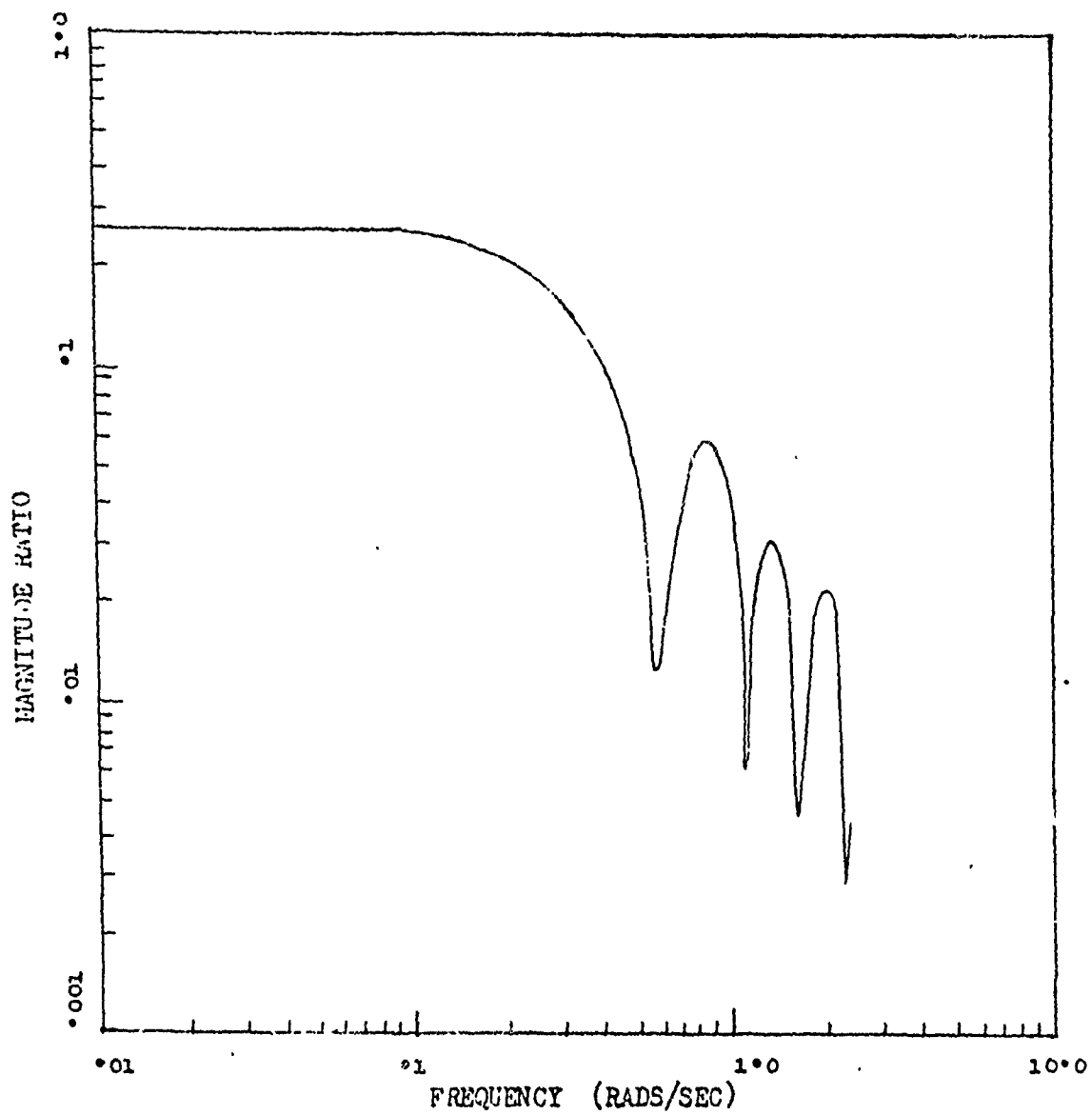
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Figure 4a. Actual frequency response of the liquid hydrocarbon temperature at outlet of exchanger.

Figure 4b. Actual frequency response of the liquid hydrocarbon
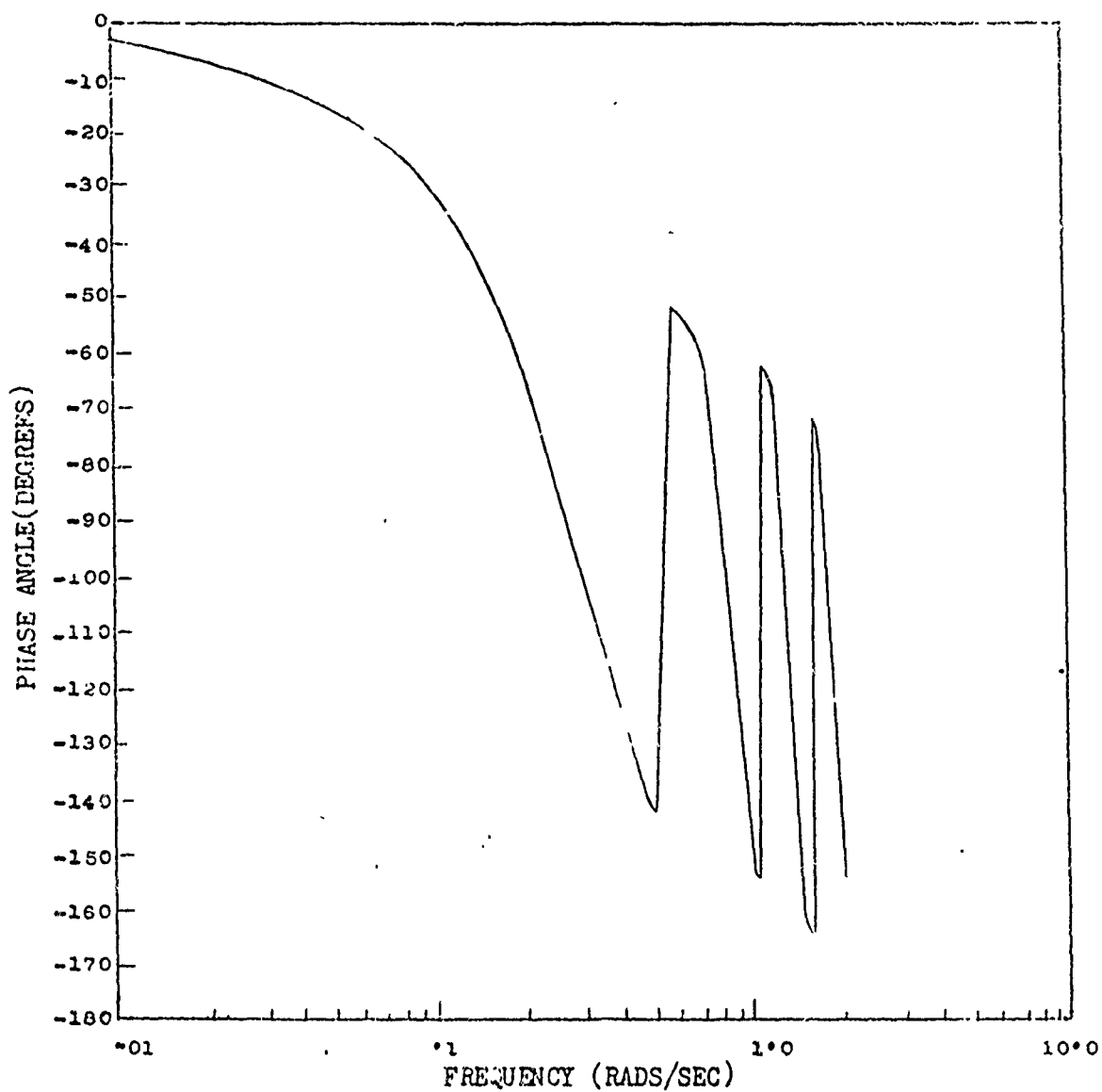
temperature at outlet of exchanger.

TABLE II

REGRESSIONS CARRIED OUT IN $G_1(s)$ FOR ANALYTICAL

LUMPED PARAMETER APPROXIMATION TECHNIQUE

Parameter	Starting Point	End Point	Sum of Squares Due To Regression	Time or Frequency Regression
t_0 τ_1	1.00 1.00	0.819612 6.43049	4.574857×10^{-2}	Time
t_0 τ_1	0.02 0.02	0 7.21836	5.959531×10^{-2}	Time
t_0 τ_1	5.00 10.00	0.820498 6.43164	4.574862×10^{-2}	Time
t_0 τ_1	10.00 1.00	0.830880 6.39516	4.575604×10^{-2}	Time
τ_1 t_0	1.00 1.00	6.11237 0.311565	2.064661×10^{-2} 7.628072	Freq
t_0 τ_1	10.00 1.00	6.11634 0.311507	2.064659×10^{-2} 7.625863	Freq
τ_1 t_0	0.01 1.00	6.11779 0.311489	2.064661×10^{-2} 7.625054	Freq
t_0 τ_1	1.00 10.00	6.11237 0.311021	2.064661×10^{-2} 7.628059	Freq

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TABLE III

REGRESSIONS CARRIED OUT IN $G_2(s)$ FOR ANALYTICAL

LUMPED PARAMETER APPROXIMATION TECHNIQUE

Parameter	Starting Point	End Point	Sum of Squares Due To Regression	Time or Frequency Regression
t_0	0.00	0	2.345726×10^{-2}	Time
τ_1	2.00	3.32047		
τ_2	2.00	3.31864		
t_0	0.00	0	2.345726×10^{-2}	Time
τ_1	0.02	3.32011		
τ_2	10.00	3.31883		
t_0	1.00	0.845206	4.627488×10^{-2}	Time
τ_1	6.00	6.45332		
τ_2	0.00	0		
t_0	1.00	0.0825281	2.405553×10^{-2}	Time
τ_1	1.00	3.28846		
τ_2	1.00	3.28648		
t_0	0.00	0	2.345726×10^{-2}	Time
τ_1	10.00	3.31984		
τ_2	10.00	3.31768		
t_0	1.00	0.0822956	2.405361×10^{-2}	Time
τ_1	10.00	3.28888		
τ_2	10.00	3.28684		

Table III (Cont'd)

Parameter	Starting Point	End Point	Sum of Squares Due To Regression	Time or Frequency Regression
t_0	0.00	0	2.052240	Time
r_1	0.02	0.02		
r_2	0.02	0.02		
r_1	10.00	3.25493	1.102068×10^{-2}	Freq
r_2	0.00	3.25379	$1.400556 \times 10^{+1}$	
t_0	0.00	0		
r_1	0.02	3.25513	1.102068×10^{-2}	Freq
r_2	1.00	3.25372	$1.400579 \times 10^{+1}$	
t_0	1.00	0		
r_1	1.00	3.25506	1.102068×10^{-2}	Freq
r_2	1.00	3.25378	$1.400578 \times 10^{+1}$	
t_0	1.00	0		
r_1	10.00	3.25539	1.102068×10^{-2}	Freq
r_2	1.00	3.25367	$1.400610 \times 10^{+1}$	
t_0	1.00	0		
r_1	3.00	3.25387	1.102067×10^{-2}	Freq
r_2	0.00	3.25374	$1.400396 \times 10^{+1}$	
t_0	0.00	0		

optima that were encountered during the time-domain regressions, and

hence a considerable disadvantage over the frequency-domain regressions

of $G_1(s)$ and $G_2(s)$. A considerable number of local optima were

encountered during the frequency and time-domain regressions on

$G_3(s)$ as shown in Table IV. It appears that the simultaneous

optimization of the dead time and time lags tend to introduce these

local optima in the response surface. The simultaneous regressions

are inherent to time domain optimizations and also were carried out

in the frequency-domain regressions of $G_3(s)$. Table V shows the

optimum parameters that were determined for both frequency and time-

domain regressions for each of the postulated transfer functions.

The frequency response of $G_1(s)$ for both frequency and time-

domain regressed parameters are shown in Figure 5. A slight

difference in the responses of the frequency and time-domain

regressed parameters was observed. Both responses gave poor

representation of the frequency response of the model equation

through the high, middle and part of the low frequencies.

The frequency response of $G_2(s)$ is shown in Figures 6 and 7.

No substantial difference in the responses of the frequency and

time-domain regressed parameters was observed. The frequency

response of $G_2(s)$ gave a better frequency approximation in the low

and part of the middle frequencies, while the high and part of the

middle frequencies still remain poorly represented.

The frequency response of $G_3(s)$ is shown in Figure 8. In this

case too, no appreciable difference in the response of the frequency

and time-domain regressed parameters was observed. $G_3(s)$ gave an

TABLE IV

REGRESSIONS CARRIED OUT IN $G_3(s)$ FOR ANALYTICAL

LUMPED PARAMETER APPROXIMATION TECHNIQUE

Parameter	Starting Point	End Point	Sum of Squares Due To Regression	Time or Frequency Regression
t_0	8.00	11.4302	2.971937×10^{-9}	Time
t_1	8.00	36.7031		
t_2	0.25	0.334522		
t_0	10.00	4.00800	3.211664×10^{-2}	Time
t_1	10.00	7.08115		
t_2	10.00	9.64756		
t_0	15.00	5.30352	3.300791×10^{-2}	Time
t_1	1.00	4.84512		
t_2	1.00	15.6808		
t_0	1.00	0.152469	2.316039×10^{-2}	Time
t_1	1.00	3.51667		
t_2	1.00	3.51452		
t_0	0.00	0	2.345726×10^{-2}	Time
t_1	3.00	3.31877		
t_2	3.00	3.32020		
t_0	20.00	11.4302	2.972004×10^{-9}	Time
t_1	5.00	36.7031		
t_2	20.00	0.334522		

Table IV (Cont'd)

Parameter	Starting Point	End Point	Sum of Squares Due To Regression	Time or Frequency Regression
t_0	20.00	11.4302	2.971918×10^{-9}	Time
τ_{-1}	20.00	36.7031		
τ_{-2}	0.02	0.334522		
t_0	0.00	0	6.626903	Freq
τ_{-1}	3.00	9.49829		
τ_{-2}	3.00	0.345235		
t_0	8.00	7.67432	$1.087615 \times 10^{+1}$	Freq
τ_{-1}	0.25	34.4549		
τ_{-2}	20.00	.0.305728		
t_0	8.00	7.67462	$1.087615 \times 10^{+1}$	Freq
τ_{-1}	5.00	34.4601		
τ_{-2}	20.00	0.305901		
t_0	20.00	19.8997	$1.925948 \times 10^{+1}$	Freq
τ_{-1}	0.25	41.8915		
τ_{-2}	8.00	0.179378		
t_0	20.00	19.8928	$1.924257 \times 10^{+1}$	Freq
τ_{-1}	20.00	42.1949		
τ_{-2}	0.25	0.170567		
t_0	20.00	21.0351	$1.088142 \times 10^{+1}$	Freq
τ_{-1}	0.25	40.6546		
τ_{-2}	0.25	0.347505		
t_0	11.00	11.4299	1.483555×10^{-5}	Freq
τ_{-1}	37.00	36.6997		
τ_{-2}	0.33	0.334694		

TABLE V

OPTIMUM PARAMETERS OF $G_1(s)$, $G_2(s)$ AND $G_3(s)$ OBTAINED IN THE

ANALYTICAL LUMPED PARAMETER APPROXIMATION TECHNIQUE

Parameters

Transfer Function	Time Domain			Frequency		
	t_o	τ_1	τ_2	t_o	τ_1	τ_2
$\dfrac{\text{Gain } \exp(-t_o s)}{(\tau_1 s+1)}$	0.8196	6.4305	--	0.3149	6.1163	--
$\dfrac{\text{Gain } \exp(-t_o s)}{(\tau_1 s+1)\,(\tau_2 s+1)}$	0	3.3205	3.3186	0	3.2539	3.2537
$\dfrac{1-\text{Sgain } \exp(-t_o s)}{(\tau_1 s+1)\,(\tau_2 s+1)}$	11.4302	36.7031	0.3345	11.4299	36.6997	0.3347
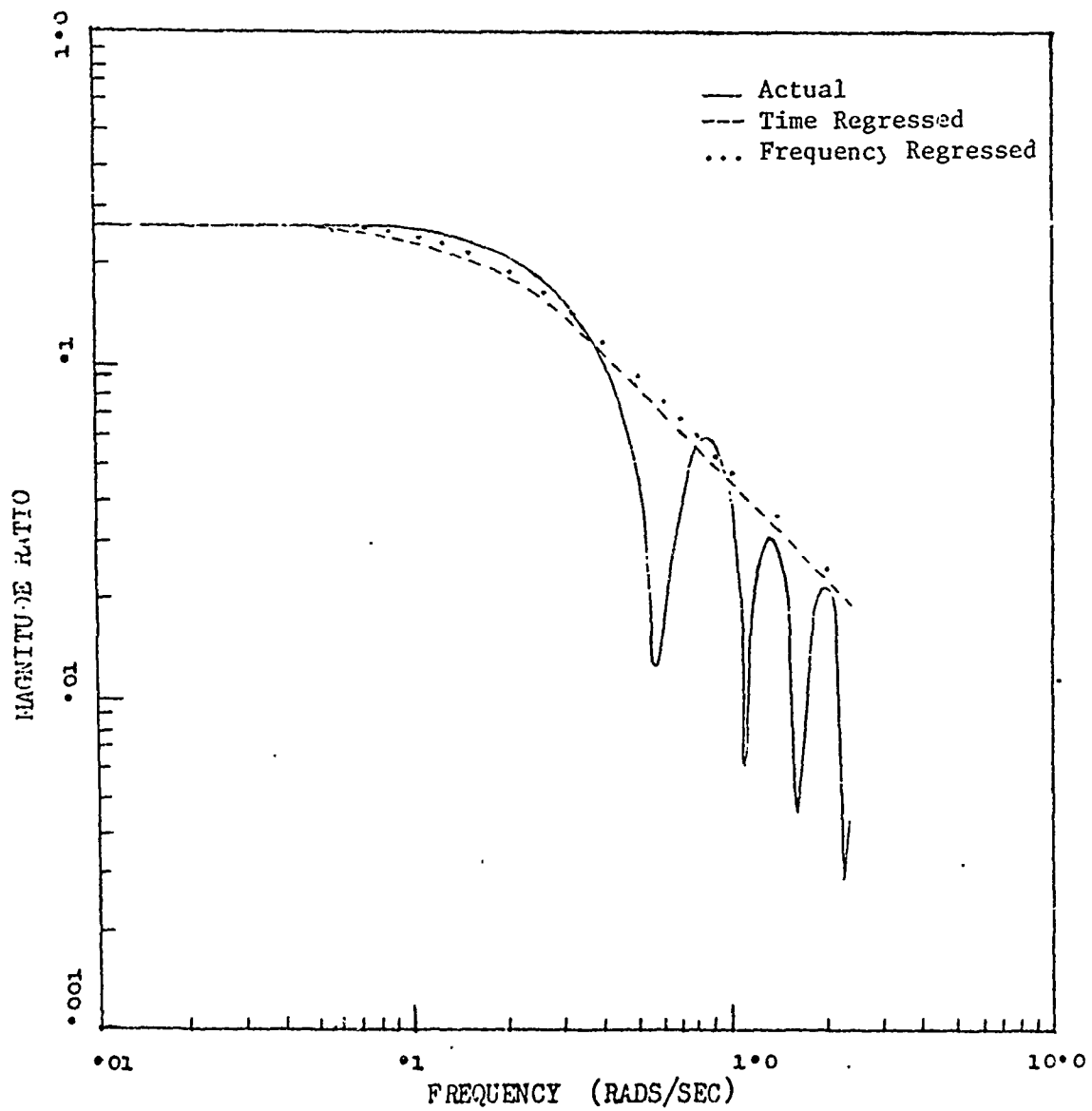
Figure 5a. Frequency response of $G_1(s)$ obtained in the analytical lumped parameter approximation technique.
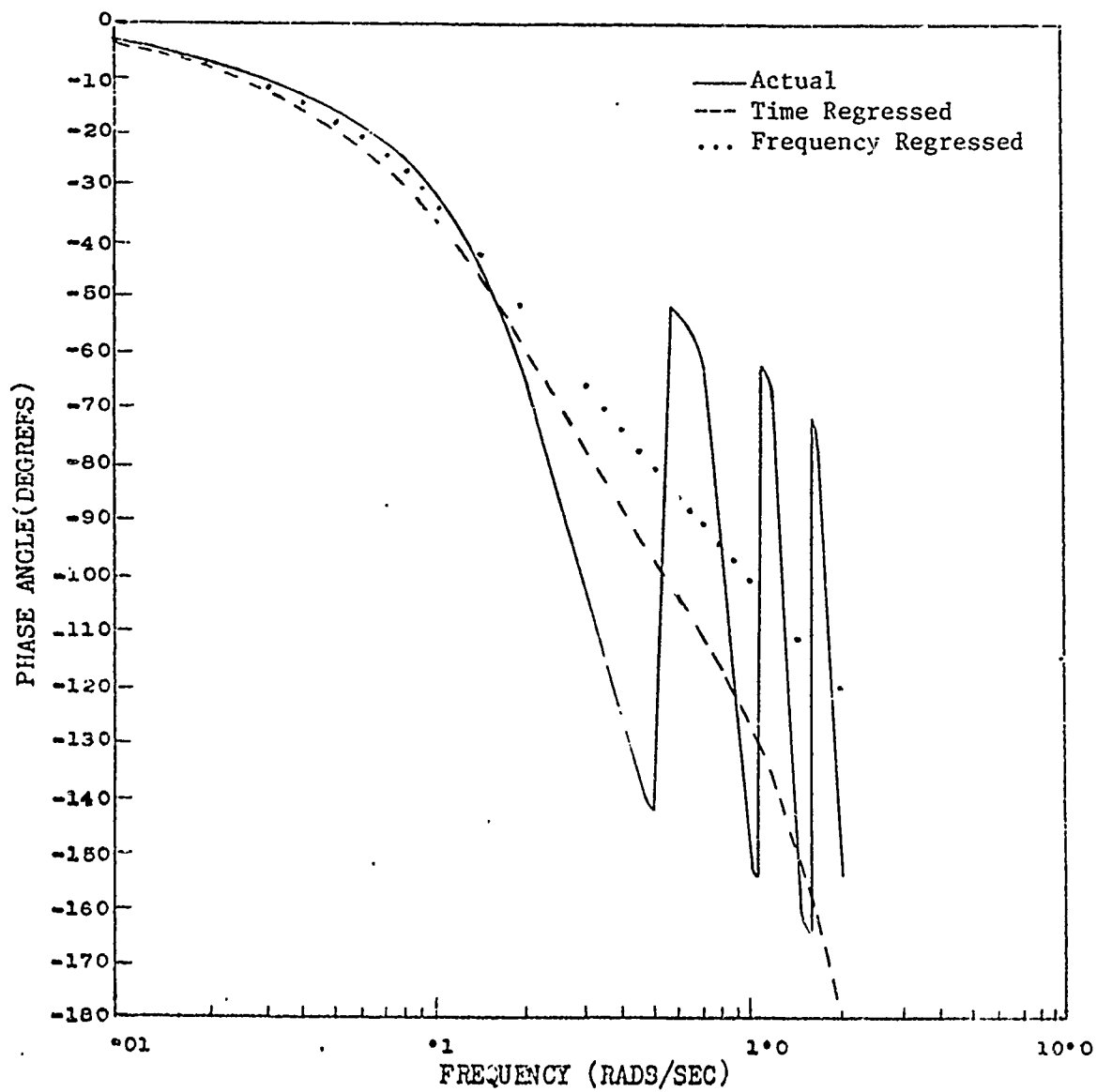
Figure 5b. Frequency response of $G_1(s)$ obtained in the analytical lumped parameter approximation technique.
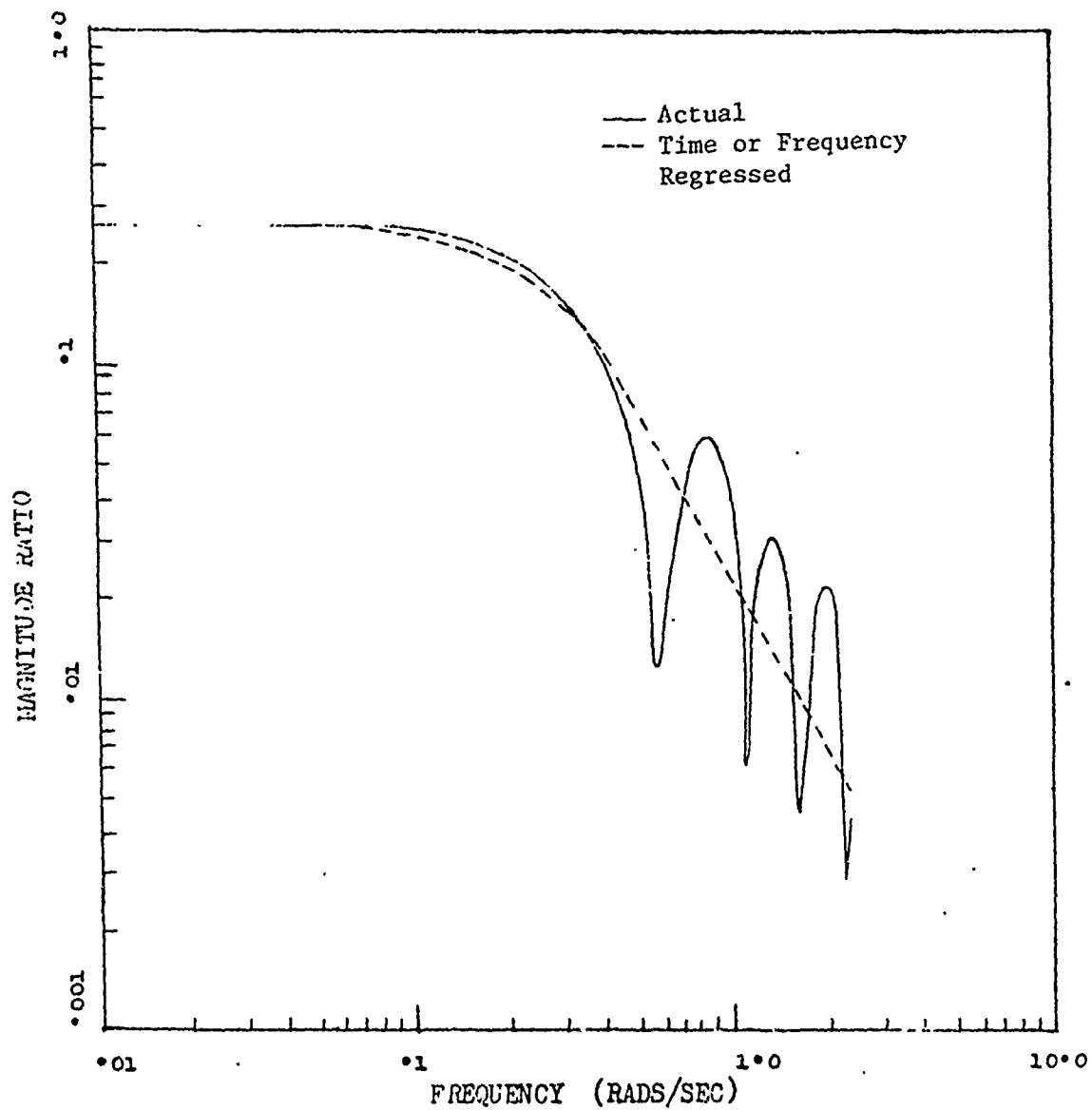
Figure 6. Frequency response of $G_2(s)$ obtained in the analytical lumped parameter approximation technique.
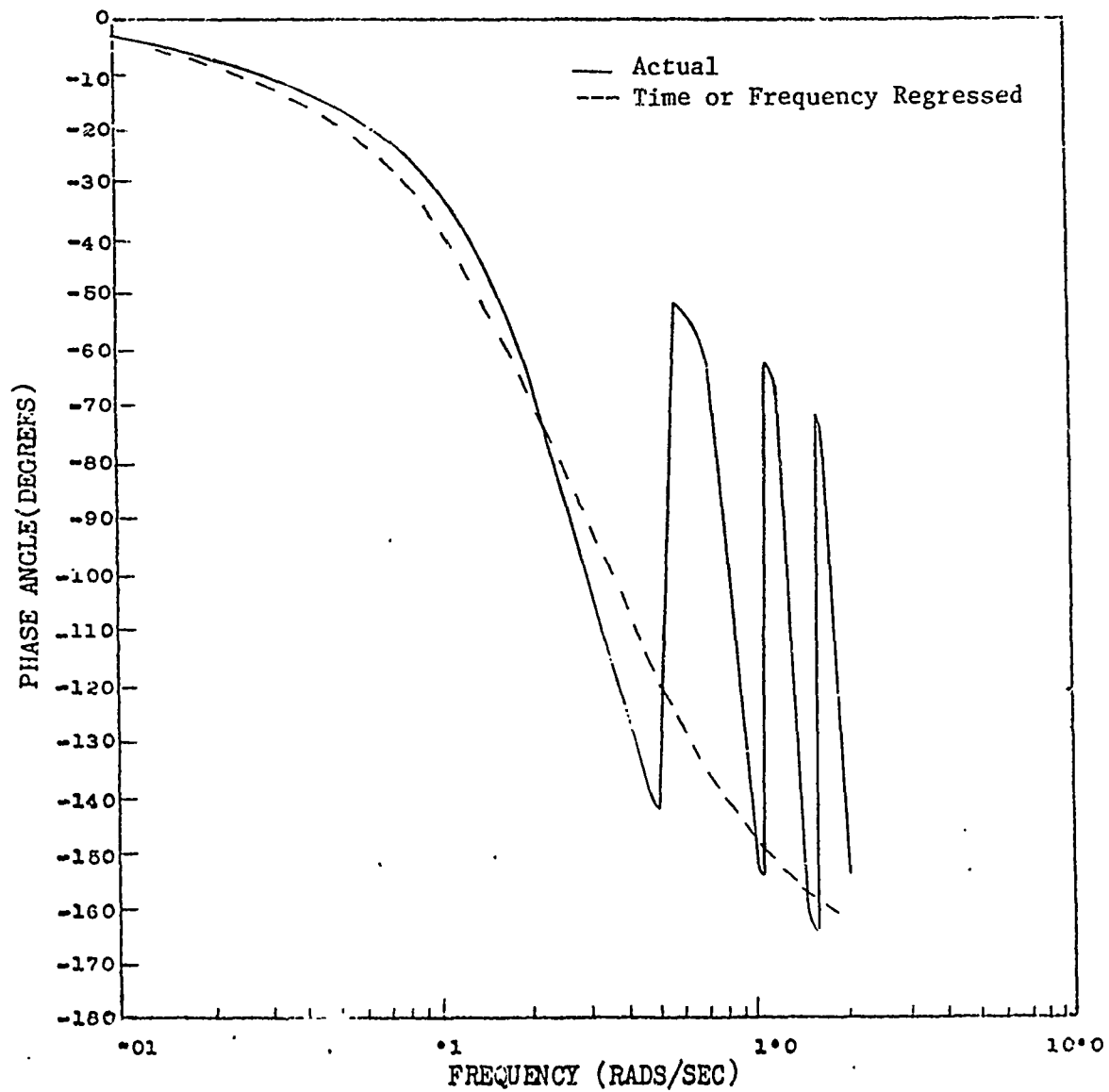
Figure 7.   Frequency response of $G_2(s)$ obtained in the analytical
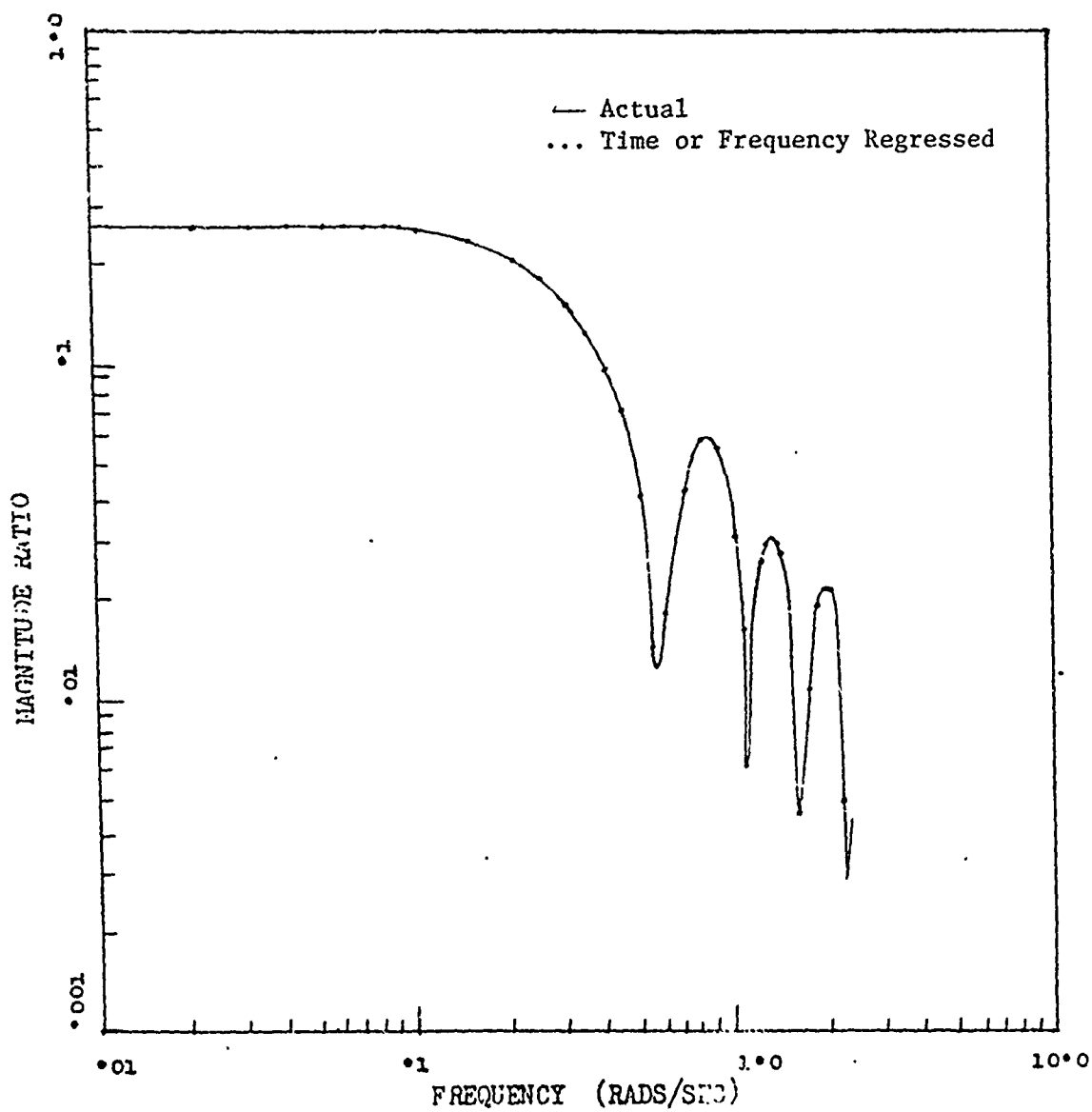
lumped parameter approximation technique.

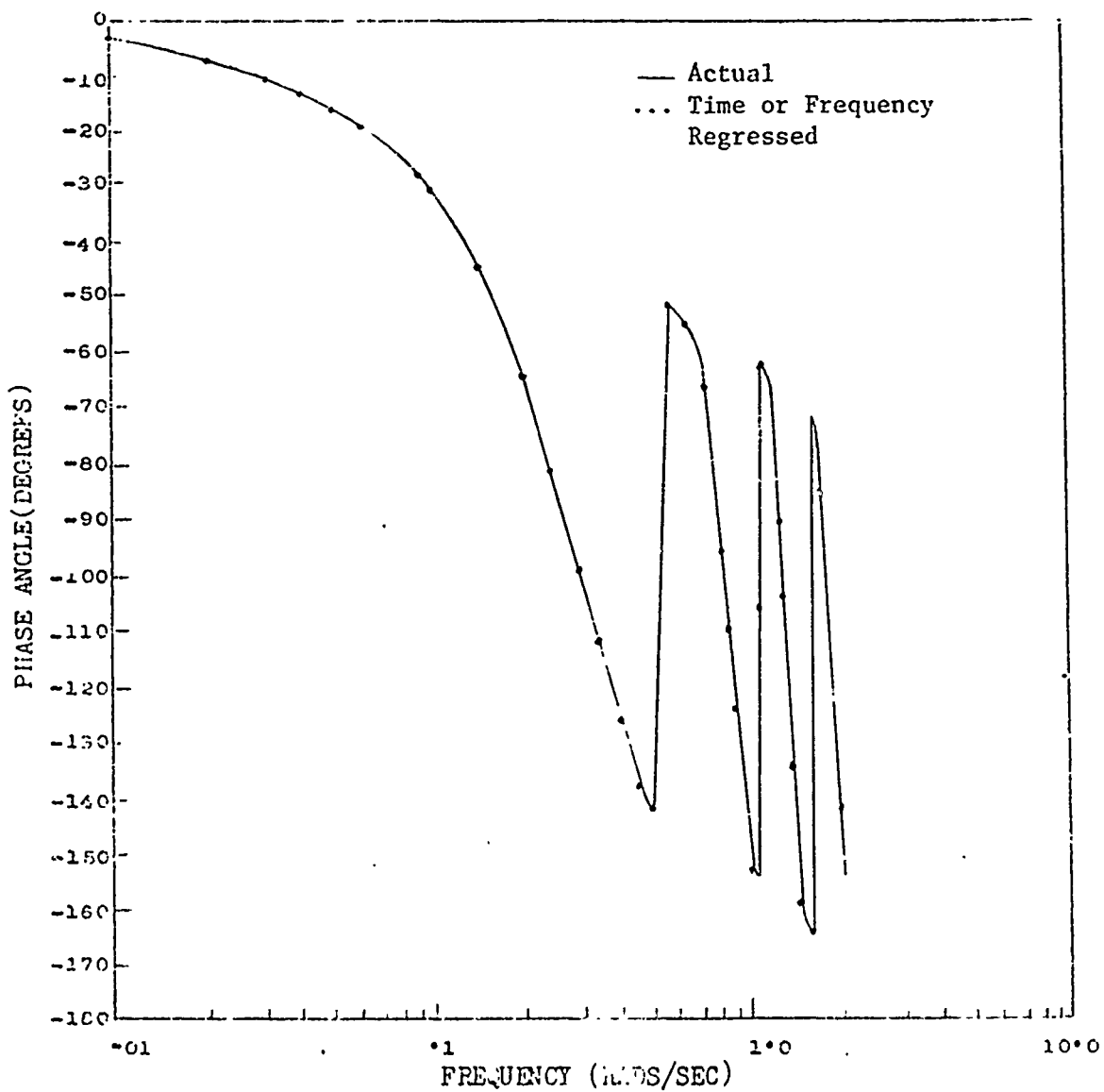Figure 8a. Frequency response of $G_3(s)$ obtained in the analytical lumped parameter approximation technique.

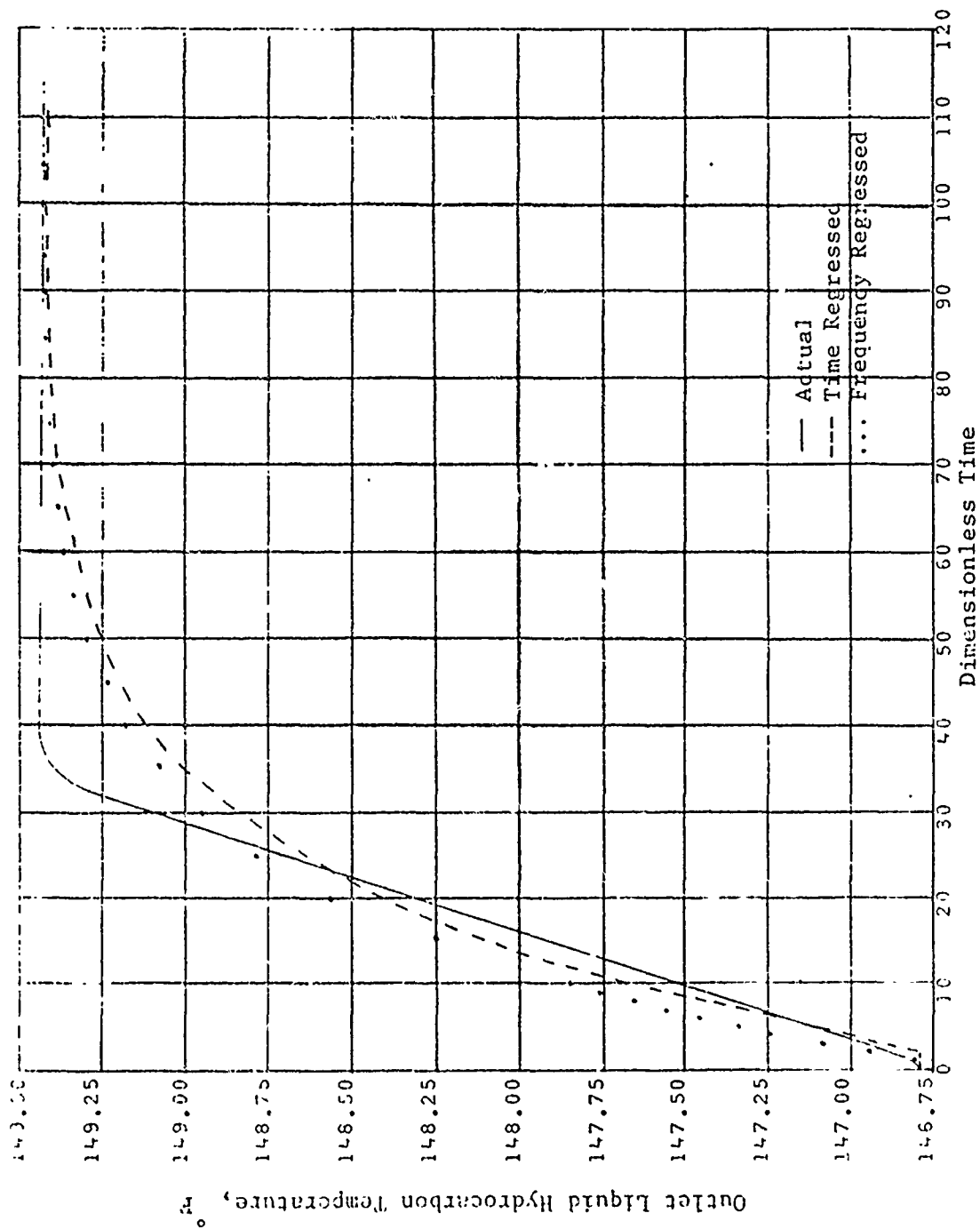Figure 8b. Frequency response of $c_3(s)$ obtained in the analytical lumped parameter approximation technique.

Figure 9. Time response of $G_1(s)$ obtained in the analytical lumped parameter approximation technique.
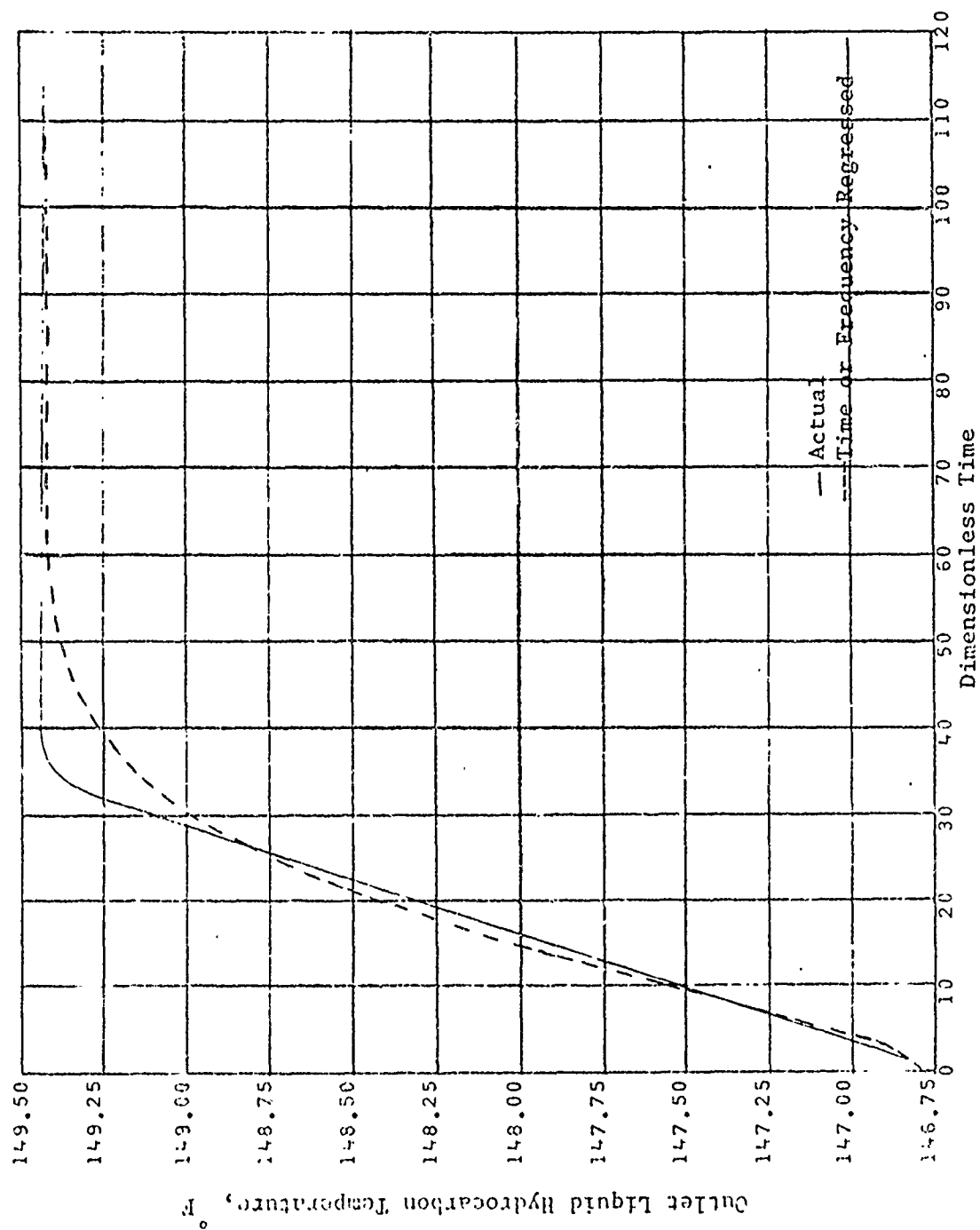
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Figure 10. Time response of $G_2(s)$ obtained in the analytical lumped parameter approximation techniques.
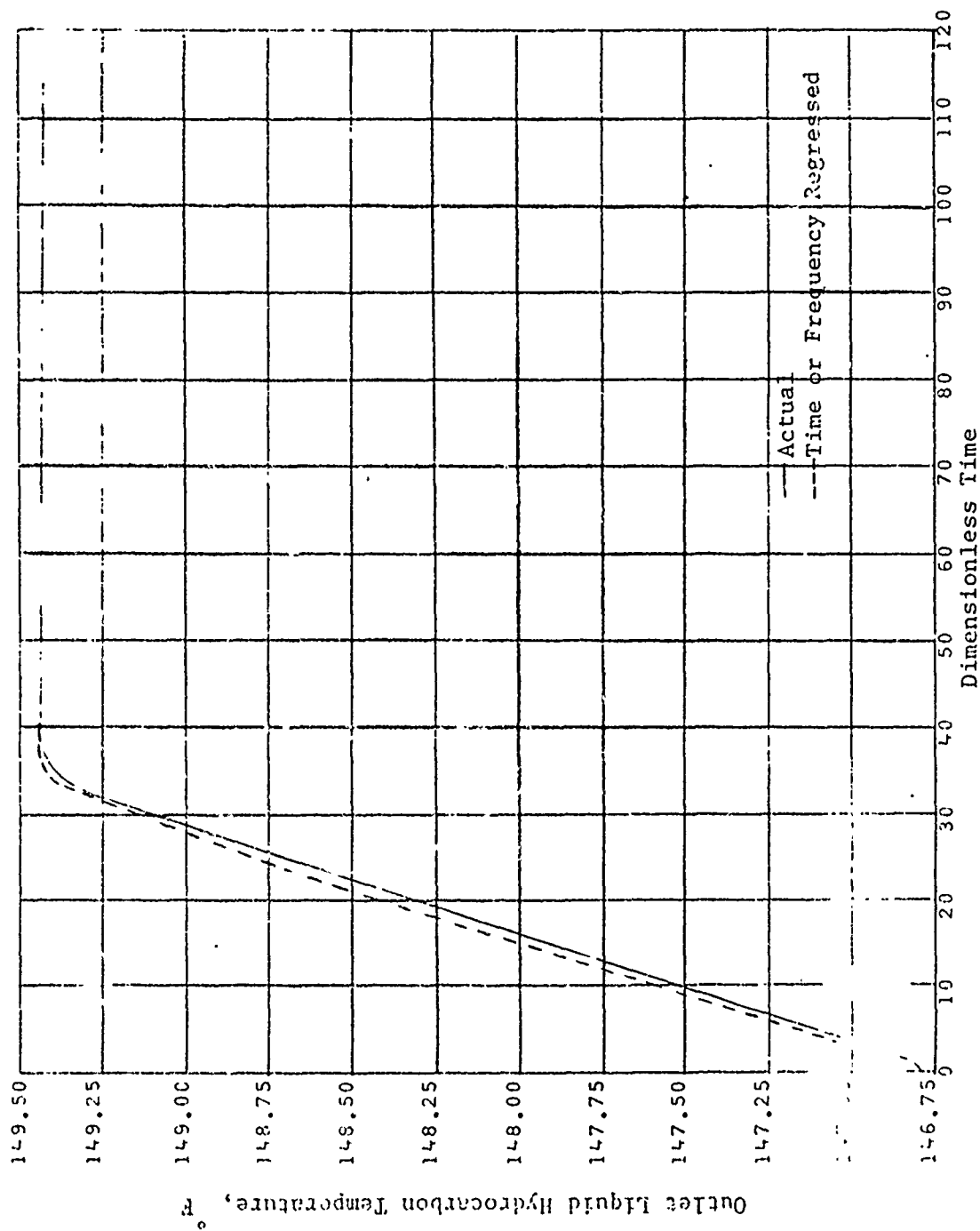
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Figure 11. Time response of $G_3(s)$ obtained in the analytical lumped parameter approximation technique.

Outlet Liquid Hydrocarbon Temperature, °F

Dimensionless Time

Actual
Time or Frequency Regressed

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almost perfect frequency representation of the model equation throughout the frequency range studied.

Time Domain Analysis of Model Equation: The time domain representation of $G_1(s)$ for both frequency and time-domain regressed parameters are shown in Figure 9. With either frequency or time-domain regressed parameters, this transfer function gives a rather poor representation of the liquid hydrocarbon temperature at the outlet of the exchanger at any given instant of time. A small difference in responses between the frequency and time-domain regressed parameters giving a slightly better approximation to the dynamics of the liquid hydrocarbon temperature.

The time domain response of $G_2(s)$ is shown in Figure 10. No difference was observed between the responses of the frequency and time-domain regressed parameters in this case. A better approximation of the liquid hydrocarbon temperature dynamics was achieved with this transfer function particularly during the first part of the solution time.

The time domain representation of $G_3(s)$ is shown in Figure 11. In this case too, no difference was observed between the responses of the frequency and time-domain regressed parameters. This transfer function provided an excellent description of the outlet liquid hydrocarbon temperature dynamics throughout the solution time.

Conclusions

Excellent representation of the time response of the liquid hydrocarbon temperature at the outlet of the exchanger was obtained with the combination of second order lag and second order lag with

dead time transfer function. Another advantage associated with this transfer function is that it only contains three parameters and can be easily implemented in analog simulation studies.

The lack of improvement over frequency regressions coupled to the additional computational effort, makes time-domain regressions impractical for convective systems.

NOMENCLATURE

A_{ic}	flow area of exchanger tubes, ft^2
A_{si}	inside surface area of exchanger tubes, ft^2/ft
A_{so}	outside surface area of exchanger tubes, ft^2/ft
C_f	Heat capacity of liquid hydrocarbon Btu/lb °F
$f(x,t)$	space dependent variable
$F(x,s)$	frequency equivalent of $f(x,t)$
$G_1(s)$	first order lag with dead time transfer function
$G_2(s)$	second order lag with dead time transfer function
$G_3(s)$	combination of second order lag and second order lag with dead time transfer function
Gain	steady state gain of system
h_i	inside heat transfer coefficient, Btu/hr(°F)ft^2
h_o	outside heat transfer coefficient, Btu/hr(°F)ft^2
L	exchanger length, ft
M_w	linear density of exchanger tubes, lb/ft
P_s	steam pressure, psig
Sgain	1-Gain
t	time, seconds
t_d	duration of input pulse, seconds
t_o	dead time, seconds
t_s	duration of output pulse, seconds
$T(x,t)$	distributed liquid hydrocarbon temperature, °F
$T_s(t)$	steam temperature, °F
$T_w(t)$	tube wall temperature, °F
$T_{ss}(x)$	steady state value of the distributed liquid hydrocarbon temperature, °F

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$(T_w)_{ss}$ steady state value of the tube wall temperature, °F

$(T_s)_{ss}$ steady state value of the steam temperature, °F

$U(y, \Psi)$ distributed liquid hydrocarbon temperature, dimensionless

v liquid hydrocarbon velocity within exchanger tubes, ft/sec

$V(\Psi)$ tube wall temperature, dimensionless

$W(\Psi)$ steam temperature, dimensionless

x distance along exchanger, ft

y distance along exchanger, dimensionless

$1(t-t_o)$ unit step applied at time $= t_o$

Greek

Ψ time, dimensionless

ω frequency, radians/second

ρ liquid hydrocarbon density, lb/ft^3

$\theta(x,t)$ distributed liquid hydrocarbon temperature representing

         deviations from steady state value, °F

$\theta_s(t)$ steam temperature representing deviations from steady

         state value, °F

$\theta_w(t)$ wall temperature representing deviations from steady

         state value, °F

$\theta(x,s)$ frequency equivalent of $\theta(x,t)$

$\theta_s(s)$ frequency equivalent of $\theta_s(t)$

$\theta_w(s)$ frequency equivalent of $\theta_w(t)$

τ time constant, seconds

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